ANGLE OF ARRIVAL ESTIMATION BY THE MATRIX PENCIL METHOD IN MOBILE SMART ANTENNAS

Farnoosh Heidari*
Department of Electrical Engineering, Bushehr Branch, Islamic Azad University, Bushehr, Iran.
*Corresponding author: E-mail: Frnsh_heidari@yahoo.com
Naser Parhizgar
Department of Electrical Engineering, Shiraz Branch, Islamic Azad University, Shiraz, Iran.
E-mail:nasserpar@yahoo.com

ABSTRACT : *In this paper, the analytical method of Matrix Pencil (MP) is evaluated for its use in tracking the Angle of Arrival (AOA) for mobile telecommunication users and the efficiency of this method is compared to the analytical method of ESPRIT. Simulations show that the MP method has less tracking errors than the ESPRIT method. Estimation by the MP method is resistant to changes in the angle of propagation and its tracking error is considerably less than the ESPRIT method, particularly when the speed of user changes which leads to non-uniform changes in the angle of propagation.*

Keywords: Estimation, Smart Antennas, ESPRIT, Matrix Pencil.

1. INTRODUCTION

Smart antennas are introduced as a desirable choice for the main stations of cellular mobile telecommunication for various reasons. The most important task of a smart antenna is to create an adequate radiation pattern in order to create the best communication channel between a main station and a user. Processing the available signals in the various elements of antenna array can lead to having access to different information about message decryption and also extracting other information from signals. One of the information that is used in many instances is the direction or angle of arrival of signals which are transmitted to antenna arrays. The angle of arrival is utilized for various purposes like radioactivity, user Geo-location etc. in order to estimate the angle of arrival of transmitted signals to antenna arrays, different methods are used, such as Kapen's minimum variance method [1], MUSIC algorithm, MUSIC root [1], [8] and maximum likelihood [1]. Most of these methods search the covered area to find the angle of propagation and as a result, spend a lot of time in finding the angle of arrival. Using methods that estimate the angle of arrival analytically without searching the area can increase the speed of estimation and decrease the complexity of calculations. The MP method is one of these methods which is used for the estimation of different parameters like the speed of user in cellular mobile telecommunication [2, 4] and the angle of arrival in CDMA [5] systems. Another analytical method that is used to estimate the angle of arrival is called ESPRIT [6]. The main advantage of the MP method over ESPRIT is its ability to estimate with less error while using a snapshot of a received signal.

In the following section, we will consider the MP algorithm. Section 3 is devoted to a brief introduction of ESPRIT. In section 4, the results of simulating angle of arrival estimation and its tracking will be presented and finally in section 5, this paper's achievements will be presented.

2 THE MATRIX PENCIL METHOD

Generally, this method is used to estimate sinusoidal signal parameters. In the case of antenna arrays, by assuming that the received signals are narrowband, the phase difference that created because of distance between elements and angle of propagation to arrays can be estimated by this method. By calculating this phase difference, the angle of propagation is revealed.

When we consider an N element array which receives M sinusoidal signals, $y_k(l)$ which is the ι-th snapshot of the received baseband signal in the k-th element of the array is as follows:

$$y_k(l) = \sum_{i=1}^{M} a_i \exp[-j2\pi kd\cos\theta_i] + n_k(l)$$
$$= \sum_{i=1}^{M} a_i z_i^k + n_k(l) \qquad k = 0,1,\ldots,N-1 \tag{1}$$

Where $a_i = u_i(l)$ is the i-th signal baseband in ι-th moment and $n_k(l)$ is the k-th base noise related to the k-th element of the array. The parameter of z_i is as follows:

$$z_i = e^{-j2\pi d\cos\theta_i} \tag{2}$$

, where θ_i is angle of arrival of the i-th signal.

First, in order to analyze the equations related to the MP method, we assume that the signals have no noise and in order to simplify the equation by using y_k instead of $y_k(l)$ and we construct the Y matrix:

$$\mathbf{Y} = \begin{bmatrix} y_0 & y_1 & \cdots & y_L \\ y_1 & y_2 & \cdots & y_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{N-L-1} & y_{N-L} & \cdots & y_{N-1} \end{bmatrix}_{(N-L)\times(L+1)} \tag{3}$$

In this equation, L is the parameter of Pencil. Y_1 and Y_2 matrices are made by removing the first and last column of Y matrix. Y_1 and Y_2 matrices can be analyzed as the following:

$$\mathbf{Y}_1 = \mathbf{Z}_L \mathbf{R} \mathbf{Z}_R \quad (4)$$

$$\mathbf{Y}_2 = \mathbf{Z}_L \mathbf{R} \mathbf{Z} \mathbf{Z}_R \quad (5)$$

In these equations:

$$\mathbf{Z}_L = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & \cdots & z_M^{N-L-1} \end{bmatrix}$$

$$\mathbf{Z}_R = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{N-L} \\ 1 & z_2 & \cdots & z_2^{N-L} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_M & \cdots & z_M^{N-L} \end{bmatrix}$$

$$\mathbf{Z} = diag\{z_1 \quad z_2 \quad \cdots \quad z_M\} \quad (6)$$

$$\mathbf{R} = diag\{a_1 \quad a_2 \quad \cdots \quad a_M\} \quad (7)$$

$\mathbf{Y}_2 - \lambda \mathbf{Y}_1$ is called the Pencil matrix and we can show that if $M \le L \le N - M$, each one of $\{z_i : i = 1,2,\cdots,M\}$ s doesn't reduce the rank of $\mathbf{Y}_2 - \lambda \mathbf{Y}_1$ Pencil matrix [3].

By choosing the L parameter in a way that it is true in the condition $M \le L \le N - M$, the generalized eigenvalues of both Y_1 and Y_2 matrices which is calculated by the equation of $\mathbf{Y}_2 = \lambda \mathbf{Y}_1$ are the same as Z_i s in equation 3. In order to calculate these values, Y_1 reverse matrix should be used. Because this matrix is not squared, its semi-reverse form ($\mathbf{Y}_1^+ = \{\mathbf{Y}_1^H \mathbf{Y}_1\}^{-1} \mathbf{Y}_1^H$) is used.

$$\lambda \mathbf{I} = \mathbf{Y}_1^+ \mathbf{Y}_2 \quad (8)$$

Where $(\bullet)^H$ indicates binary transposition and I is the identity matrix. Therefore, in order to calculate angles of arrival (AOA), we need to calculate the values of $\{z_i : i = 1,2,\cdots,M\}$, and also estimate $\theta_i \quad \{i = 1,2,\cdots,M\}$ s which are the angles of arrival by using equation (3).

If the data has ($y_k(l)$) noise, first a kind of filtering is done on Y data matrix in order to separate signal and noise area and only the signal area data be used for estimation [2,3 , 5].

3. THE ESPRIT METHOD

This method also like the MP method estimates the signal arrival by estimating the phase difference between the received signals in various elements of the array which are a result of the distance between elements and the angle of arrival. In this method, N-1 is considered a pair of the element and estimation is done by creating two vectors; one of which includes M signals transmitted to the first elements $x_1(t)$ and the other one includes M signals transmitted to the second elements $x_2(t)$ of this pair of sensors.

$$\mathbf{x}_1(l) = [y_0(l) \quad y_1(l) \quad \cdots \quad y_{N-2}(l)]^T$$
$$\mathbf{x}_2(l) = [y_1(l) \quad y_2(l) \quad \cdots \quad y_{N-1}(l)]^T \quad (9)$$

In the absence of noise these equation will be as follows:

$$\mathbf{x}_1(l) = \mathbf{A}\mathbf{s}(l)$$
$$\mathbf{x}_2(l) = \mathbf{A}\mathbf{\Phi}\mathbf{s}(l) \quad (10)$$

The A matrix with the dimension of $(N-1) \times M$ is known as the array propagation matrix and the elements of row p and column q are calculated by the following equation.

$$p = 1,2,\cdots,N-1 \quad , \quad q = 1,2,\cdots,M \qquad A_{pq} = z_q^{p-1} \quad (11)$$

, And z_q is calculated by equation (3).

$$\mathbf{s}(l) = [u_1(l) \quad u_2(l) \quad \cdots \quad u_M(l)]^T \quad (12)$$

$\mathbf{\Phi}$, is a diagonal M×M matrix with the main diameter z_i elements (in equation 3) which determines the phase difference of signals received by the pair of sensors.

Now we create X_1 matrix with N-1 rows and n columns which includes $x_1(l)$ data (n is the number of snapshots). We create X_2 matrix with $x_2(l)$ data by using the same method. This kind of data collection is called Rectangular Windowing.

$$\mathbf{X}_1 = [\mathbf{x}_1(l) \quad \mathbf{x}_1(l+1) \quad \cdots \quad \mathbf{x}_1(l+n-1)]$$
$$\mathbf{X}_2 = [\mathbf{x}_2(l) \quad \mathbf{x}_2(l+1) \quad \cdots \quad \mathbf{x}_2(l+n-1)] \quad (13)$$

We will ultimate create the X matrix as the following:

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}\mathbf{\Phi} \end{bmatrix} \mathbf{S}$$
$$\mathbf{S} = [\mathbf{s}(l) \quad \mathbf{s}(l+1) \quad \cdots \quad \mathbf{s}(l+n-1)] . \quad (14)$$

By analyzing this we will have singular values.

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{W}^H \quad (15)$$

After analyzing the singular values on X matrix, we will create U matrix like the following:

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_3 \\ \mathbf{U}_2 & \mathbf{U}_4 \end{bmatrix} \quad (16)$$

U_1 and U_2 matrices have N-1 rows and M columns. By using these two matrices, we create the new $\tilde{\mathbf{U}}$ matrix which is $\tilde{\mathbf{U}} = [\mathbf{U}_1 \quad \mathbf{U}_2]$. By analyzing the singular values on $\tilde{\mathbf{U}}$ which is $\tilde{\mathbf{U}} = \mathbf{T}\mathbf{\Lambda}\mathbf{V}^H$ and dividing it by V we will have:

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & \mathbf{V}_3 \\ \mathbf{V}_2 & \mathbf{V}_4 \end{bmatrix} \quad (17)$$

Both V_1 and V_2 matrices have M×M dimensions. It is proved that the generalized eigenvalues (λ_k) of both V_1 and V_2 matrices which resulted from an equation similar to equation 11 are the same as

$\{z_i : i = 1,2,\cdots,M\}$

in equation 3 [6]. Therefore, the angle of arrival is estimated without searching the area.

We should remind you that the MP method estimates the angle of arrival by a snapshot, while ESPRIT need n snapshots of signals which are received by the array.

4. THE RESULTS OF SIMULATION

In this section we will consider the efficiency of the MP method and compare its results with the ESPRIT method. Consequently, a linear antenna array system with seven antennas was considered, in a way that the distance between them is half the wavelength. In addition, the Pencil parameter is L=4 in the MP method. It should be explained that in order to estimate by the MP method with n snapshots, the average of n time estimations is considered. In the first state, the location of users is fixed and 3 signals are received by the array with angels of 35, 75 and 140 degrees. In order to investigate the efficiency of the MP algorithm and compare it with SPRIT, we evaluate the behavior of the 75 degree angle estimation as a sample and in order to statistically investigate the results, the estimation is done 1000 time by both methods. In figure 1, variance of error graphs are drawn for both methods for various snapshot with the ratio of signal to noise. It is observed that the ESPRIT method has difficulties in estimating one sample but it is better than the MP method in estimating more samples while users' location is fixed.

In the second state, in order to investigate the efficiency of the MP method and compare it to ESPRIT, user tracking is considered. In this state, the communication channel is a junction with three assumed trajectories. In figure 2, the trajectory related to the angle of arrival (AOA) and also the manner user tracking in estimation by the methods of MP and ESPRIT is shown while the change of angle is slow. For statistical comparison in figure 3, the Root Mean Squares Error (RMSE) is drawn for 100 times estimation of user's angle of arrival with slow angle change. As shown, the MP method has less errors compared to the ESPRIT method.

The findings indicate that the MP method is better in user tracking compared to the ESPRIT method.

5. CONCLUSION

Angle of arrival estimation and user tracking by the MP and ESPRIT method were evaluated and compared in this paper. The MP method estimates the angle of arrival with one snapshot while ESPRIT needs n snapshots of signals received by the array. Simulations show that if the angle of arrival is the same in these snapshots, estimation works better with the ESPRIT method when we use more snapshots. But, when one snapshot of the signal is available, the MP method is more efficient. When tracking mobile users whose angle of propagation changes with time, the MP method is more efficient than the ESPRIT method because it uses one

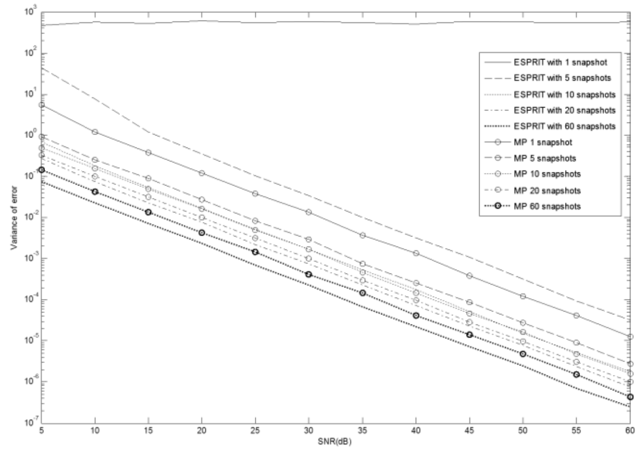

Fig. 1 Comparing the MP and ESPRIT variance of error with different number of snapshots.

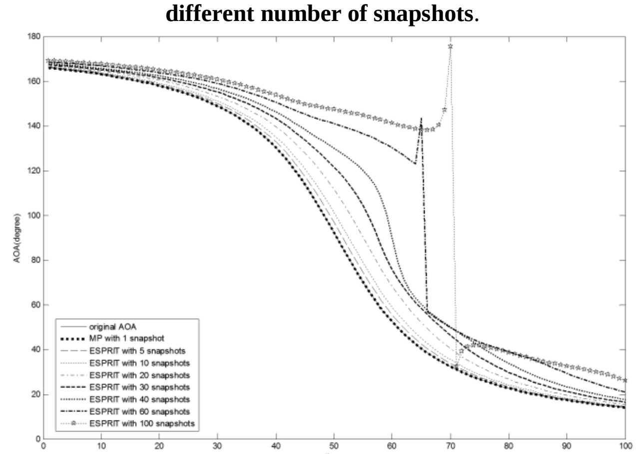

Fig. 2 The manner of tracking by MP and ESPRIT with low user speed (the trajectory of one time tracking).

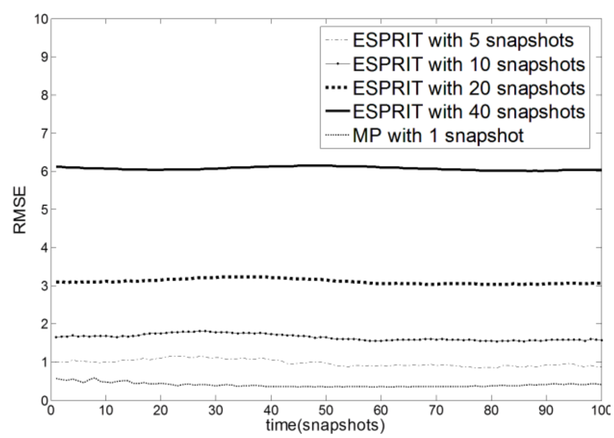

Fig. 3 Root Mean Squares Error in 100 times user tracking with low user speed. MP with one snapshot and ESPRIT with different number of snapshots.

snapshot of the signal for estimation. Furthermore, the MP method is independent of user's speed change. Ultimately, the MP method is a desirable choice for tracking mobile telecommunication users.

REFERENCES

[1] J. C. Liberti, and T. S. Rappaport, Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications, *Prentice Hall*, 1999.

[2] Y. Hua and T. K. Sarkar, "Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise," *IEEE Transaction on Acoustic, Speech and Signal Processing*, **vol. 38**, no. 5, pp. 814-824, May 1990.

[3] T. K. Sarkar and O. Pereira, "Using the matrix pencil method to estimate the parameters of a sum of complex exponentials," *IEEE Antenna and Propagation Magazine*, **vol. 37**, no. 1, pp. 48-55, February 1995.

[4] M. Turkboylari and L. Stuber, "Eigen-matrix pencil method-based velocity estimation for mobile cellular radio systems," *ICC 2000 - IEEE International Conference on Communications*, no. 1, pp. 690-694, June 2000.

[5] C. K. E. Lau, R. S. Adve and T. K. Sarkar, "Combined CDMA and matrix pencil direction of arrival estimation," *2002 Proceedings IEEE 56th Vehicular Technology Conference,* VTC. 2002-Fall, pp. 496-499, 2002.

[6] A. Paulraj, R. Roy and T. Kailath, "Estimation of signal parameters via rotational invariance techniques-ESPRIT," *Proc. of 19th Asilomar Conf. on Circuit and Systems*, pp. 83-89, 1985 **.**

[7] T. K. Sarkar, M. C. Wicks, M. S. Palma and R. J. Bonneau, Smart Antennas, *John Wiley & Sons*, Inc 2003.

[8] S. U. Pillai, C. S. Burrus, Array signal processing, *Springer-Verlag New York* Inc, 1989.